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LAST ORBITS OF BINARY BLACK HOLES*

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Binary black hole systems in the pre-coalescence stage are numerically constructed by demanding that the associated spacetime admits a helical Killing vector. Comparison with third order post-Newtonian calculations indicates a rather good agreement until the innermost stable circular orbit.

Keywords: Black holes; binary systems; gravitational radiation

1. Quasi-equilibrium configurations of binary black holes

1.1. *Helical Killing vector*

Binary black holes are considered as one of the most promising sources for the gravitational wave interferometric detectors LIGO, VIRGO, GEO600 and TAMA currently under construction, as well as for the space interferometer LISA in project. From the theoretical point of view, computing the final inspiral and merger of binary black holes has proved to be a very difficult task. However, when the black holes are far enough so that the radiation reaction timescale is large as compared to the dynamical timescale, the orbits can be approximated as closed ones. Moreover, due to the accumulated effects of the reaction to gravitational radiation, these orbits can be considered as circular.

In such a case (closed circular orbits), the spacetime possesses a one-parameter symmetry group, whose integral curves are helices. The associated Killing vector is called the *helical Killing vector*. In full general relativity, requiring the helical symmetry for a binary system imply equal amounts of outgoing and incoming grav-

*This talk is dedicated to the memory of our friend and colleague Jean-Alain Marck.

itational radiation.[1] The spacetime cannot therefore not be asymptotically flat.[2] However, there are two interesting cases where helical symmetry and asymptotic flatness are compatible: (i) in the post-Newtonian (PN) approximation, up to the second order and (ii) the Isenberg-Wilson-Mathews (IWM) approximation of general relativity (see e.g. Sec. IV.C of Ref. [3]), which assumes that the spacetime can be foliated by a family of conformally flat spacelike hypersurfaces and reduces the Einstein equations to five elliptic equations. Both approximations (i) and (ii) do not allow for gravitational radiation, which explains why they permit asymptotically flat spacetimes. They are valid approximations to general relativity when the gravitational radiation content of spacetime does not play any important dynamical role. Note that the IWM formulation contains the first order PN one.

In our first study of the binary black hole problem,[4,5] we have used the helical symmetry along with the IWM approximation

1.2. *Rigid rotation*

Regarding the rotation state of the black holes, we have chosen each black hole to be corotating (synchronized binary), so that the whole system is in rigid rotation. There are several motivations for this: (i) It has been recently argued[6] that when the black holes are close together, tidal forces may lead to an efficient transfer of spin angular momentum to orbital angular, which might lead to synchronization. However this not clear yet. (ii) As discussed by Friedman et al.[3], corotating black holes are the only state compatible with the helical symmetry in full general relativity (basically, any other rotation state would result in shear and increase of the horizon area, which is not allowed by the helical symmetry). Detweiler[1] has already pointed this by noticing that gravitational radiation going down in the hole from the distant past piles up near the distant horizon, resulting in a divergence of the metric, except for the corotating state. (iii) For the IWM spacetimes we are considering, other rotation states are permitted,[7] but the corotating one has a very simple geometrical interpretation: the black holes horizons are in this case Killing horizons, i.e. the helical Killing vector is tangent to the black hole horizons.[4]

1.3. *Determination of the orbital angular velocity*

In the IWM framework, the field equations reduce to five elliptic equations and the orbital angular velocity Ω appears only in the boundary condition at infinity for the corotating shift vector. This contrasts with the fluid case, where Ω shows up in the equation governing the equilibrium of the fluid. We have determined Ω by demanding that the ADM mass be equal to the Komar-like one[4], a requirement which reduces to the classical virial theorem at the Newtonian limit. We have verified that when the holes are far apart, this procedure leads to Kepler's third law.

The minimum of ADM mass along a sequence of decreasing circular orbits with constant horizon area marks the limit of orbital stability.[3] We call this configuration

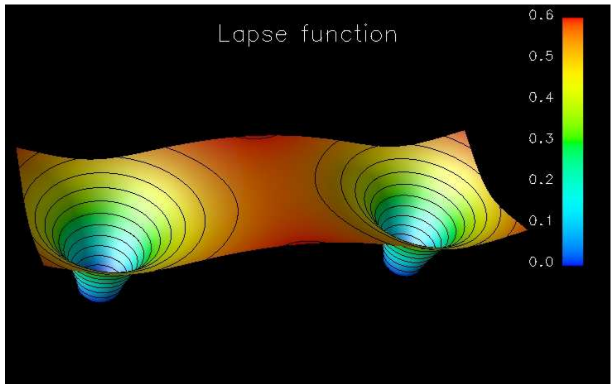

Fig. 1. Lapse function in the equatorial plane for the ISCO configuration.

the ISCO (for *Innermost Stable Circular Orbit*).

2. Numerical solution

The five non-linear elliptic equations describing corotating binary black holes in the IWM approximation have been solved by means of a multi-domain spectral method.[8,9,10] We use spherical coordinate systems centered on each black hole, which allow for an easy implementation of the boundary conditions on the horizons. Moreover the computational domain extends to spacelike infinity, thanks to some suitable compactification of the external domain. This permits a rigorous implementation of the boundary condition at infinity (flat spacetime). The numerical implementation has been performed by means of an object-oriented code based on the Lorene library.[11] Numerous tests of the code have been presented in Ref.[5].

3. Comparison with post-Newtonian results

Prior to our study[4,5], the agreement between numerical computations of orbiting black holes and analytical (post-Newtonian) ones was very bad (compare the dotted line with the dot-dashed one in Fig. 2a). Notably the orbital frequency at the ISCO was differing by a factor of two (compare the triangles with the circles and diamonds in Fig. 2b). Our results, based on the helical killing vector approach, turned out to be much closer to the post-Newtonian ones (compare the dark solid line with the light solid one in Fig. 2a, as well as the square with the circles and diamonds in Fig. 2b). We present a detailed comparison with the Effective One Body (EOB) analytical approach in Ref. [15].

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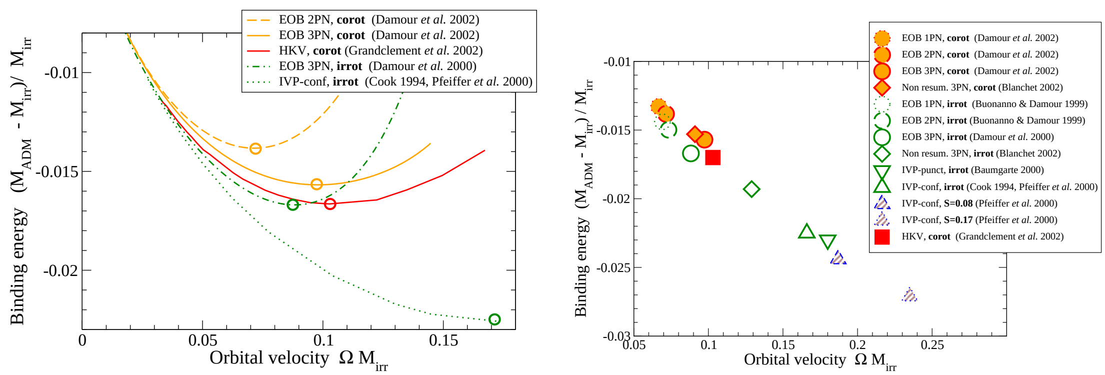

Fig. 2. Comparison between numerical results and post-Newtonian ones: *left:* binding energy along a constant area sequence of binary black holes. The numerical results are our helical killing vector approach (HKV) [5], and IVP, the initial value approach with effective potential[12,13]; *right:* Values of the binding energy and orbital frequency at the ISCO for various numerical[12,13,14,5] and post-Newtonian[15,16,17,18] methods.

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